

Inference at *
of proof for Lemma order_split:

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⊢∀T:Type, R:(T→T→ ℙ).
  Order(T;x,y.R(x,y))
  ⇒ (∀x, y:T. Dec(x = y))
  ⇒ (∀a, b:T. R(a,b) ⇔ (strict_part(x,y.R(x,y);a;b) ∨ (a = b)))
  by (((Unfold 'strict_part' 0)
  CollapseTHEN (AGenRepD ["compound";"basic"])).)

  CollapseTHENA ((Auto_aux (first_nat 1:n) ((first_nat 1:n),(first_nat 3:n)) (first_tok
:t) inil_term))).
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1:

1. $T : \text{Type}$
 2. $R : T \rightarrow T \rightarrow \mathbb{P}$
 3. $\forall a:T. R(a,a)$
 4. $\forall a, b, c:T. R(a,b) \Rightarrow R(b,c) \Rightarrow R(a,c)$
 5. $\forall x, y:T. R(x,y) \Rightarrow R(y,x) \Rightarrow (x = y)$
 6. $\forall x, y:T. \text{Dec}(x = y)$
 7. $a : T$
 8. $b : T$
 9. $R(a,b)$
- $\vdash (R(a,b) \ \& \ (\neg R(b,a))) \vee (a = b)$

2:

1. $T : \text{Type}$
 2. $R : T \rightarrow T \rightarrow \mathbb{P}$
 3. $\forall a:T. R(a,a)$
 4. $\forall a, b, c:T. R(a,b) \Rightarrow R(b,c) \Rightarrow R(a,c)$
 5. $\forall x, y:T. R(x,y) \Rightarrow R(y,x) \Rightarrow (x = y)$
 6. $\forall x, y:T. \text{Dec}(x = y)$
 7. $a : T$
 8. $b : T$
 9. $(R(a,b) \ \& \ (\neg R(b,a))) \vee (a = b)$
- $\vdash R(a,b)$